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TECHNICAL REPORT RD-GC-86-17

OPEN-LOOP INERTIAL MEASUREMENT UNIT ERROR ANALYSIS MONTE CARLO AND ROOT-SUM-SQUARE

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SEPTEMBER 1986

U.S. ARMY MISSILE COMMAND
Redstone Arsenal, Alabama 35898-5000

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This report defines closed-loop and open-loop error analysis techniques commonly used in the aerospace industry. Equations for modeling inertial platform error sources (including accelerometers, gyros, and alignments) are also given. Correlation of data obtained by Monte Carlo techniques is compared to data generated by root-sum-square methods and shown for a typical ballistic missile. Finally, validation of the Open-Loop Error Analysis Program is verified by comparison to independent error analysis program (GEAP), and to an error covariance program. In each of these cases the results have proven comparable serving to validate the results.

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I. INTRODUCTION

The purpose of this report is to document the computer program used in the Guidance and Control Directorate of the Research, Development, and Engineering Center to evaluate Inertial Measurement Unit (IMU) performance. This program, resident on the VAX 11-780, was validated using PERSHING input data but is applicable for any inertial system. The validation results were obtained using both Root-Sum-Square and Monte Carlo statistical techniques. The results compared favorably to the available reference data, to the analytical results, and to the error covariance techniques.

For this report, an IMU is considered to be an instrument package containing three orthogonally-mounted accelerometers and three orthogonally-mounted gyroscopes. The package may be gimballed or strapdown.

The approach used in this "Open-Loop Error Analysis" procedure is to first generate a nominal (error-free, reference) missile trajectory, based on a reference acceleration profile as a function of time for the x, y, and z directions. This acceleration profile can be easily changed to represent different trajectories, as desired. The program interpolates between tabulated values of times and accelerations when necessary to determine the specific velocities and positions needed at specific times. These obtained values are then used to determine the range, the altitude, and the angle of approach of the missile.

Initially, the program is run with the ideal trajectory and inertial platform error coefficients given values of zero (i.e., a perfect platform). Next, errors are incorporated into the trajectory from an IMU error budget and the corrupted system is flown. Then, position errors are calculated by taking the difference of the positions, at any time during flight between the trajectory, from the ideal (perfect) system and the trajectory using the corrupted system.

II. THE INERTIAL SYSTEM

A. The Basic Function of The Inertial System

The purpose of an inertial guidance system is to place some vehicle at a predetermined aim point. The target may be located at some other point on the earth, at some point above the surface of the earth, or at any defined arbitrary point. Guidance system accuracy is evaluated in terms of the missile's ability to intercept the target position and sometimes with a required velocity vector. The error analysis procedures evaluate a portion of the guidance system accuracy by analyzing the effects that the inertial measurement errors have on the accuracy of the missile to hit the target. These evaluations can then be used to optimize the guidance system design. The basic difference between the inertial guidance and other guidance systems is that inertial guidance is completely self-contained. This feature is valuable to the military because of its independence from weather and radar jamming. The inertial navigation system is basically a system whose input is a physical acceleration vector integrated by the flight computer to obtain vehicle velocity and position for its output.

An IMU normally contains three precision accelerometers and three gyroscopes mounted so that optimum missile performance is obtained and target miss is reduced. Figure 1 shows a gimballed platform using single-axis gyros.

The accelerometer's primary function is to measure all components of missile acceleration from liftoff to impact. Initially, three accelerometers are aligned to a known reference frame and mounted with their input axes mutually perpendicular so that any acceleration of the missile is resolved along these three axes. The basic form of the accelerometer can be thought of as including a mass of known values supported in a frictionless slide bearing and restrained by a spring which obeys Hooke's law. Upon acceleration of the base, the spring must supply a force to restrain the mass and spring deflection is taken as a measure of acceleration. In practice, the proof mass is constrained to be essentially at the pickoff null position by the servo loop. The current required to slave the proof mass to null is proportional to acceleration. Figure 2 shows the basic concept of accelerometer behavior.

The gyroscope's primary function is to provide and maintain a reference orientation for the accelerometers. A gyro may be thought of as a rapidly spinning rotor of substantial moment of inertia, supported on some kind of mount which allows freedom of tilt of the spin axis relative to the base on which it is mounted. The rotor may be mounted either by gimbals or by a ball and socket joint. For simplicity, assume that the mount is free from friction and other kinds of resistance to angular motion. When such a device is initially set with the spin axis pointed in a selected direction, the spin axis preserves such direction. Gyros are classified as two-degree-of-freedom, or two-axis gyros, and one-degree-of-freedom, or single-axis gyros, depending on the number of degrees of freedom of tilt of the rotor relative to the base, not counting the spin-axis freedom. Figure 3 shows the construction of a single-axis gyro.

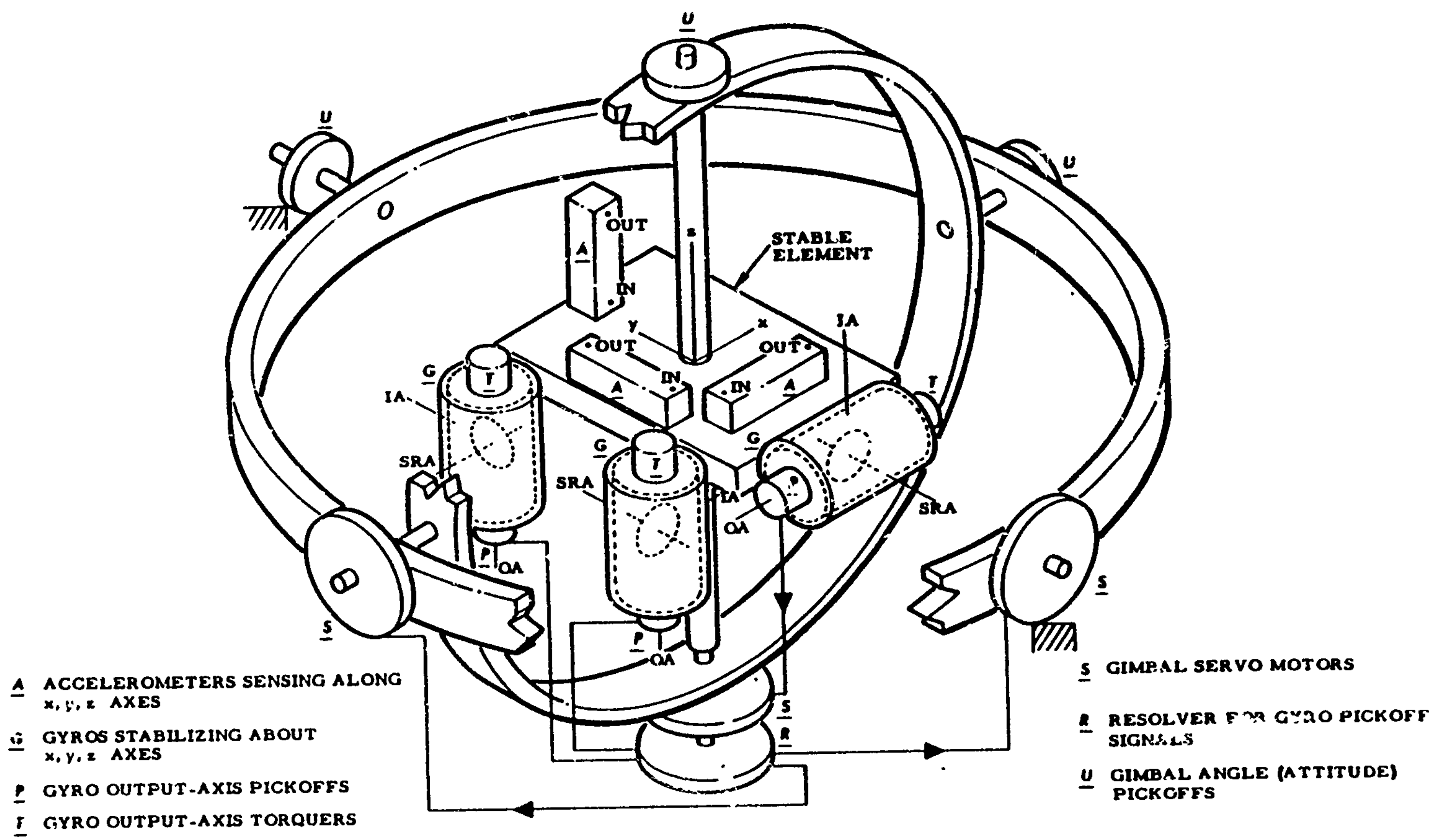
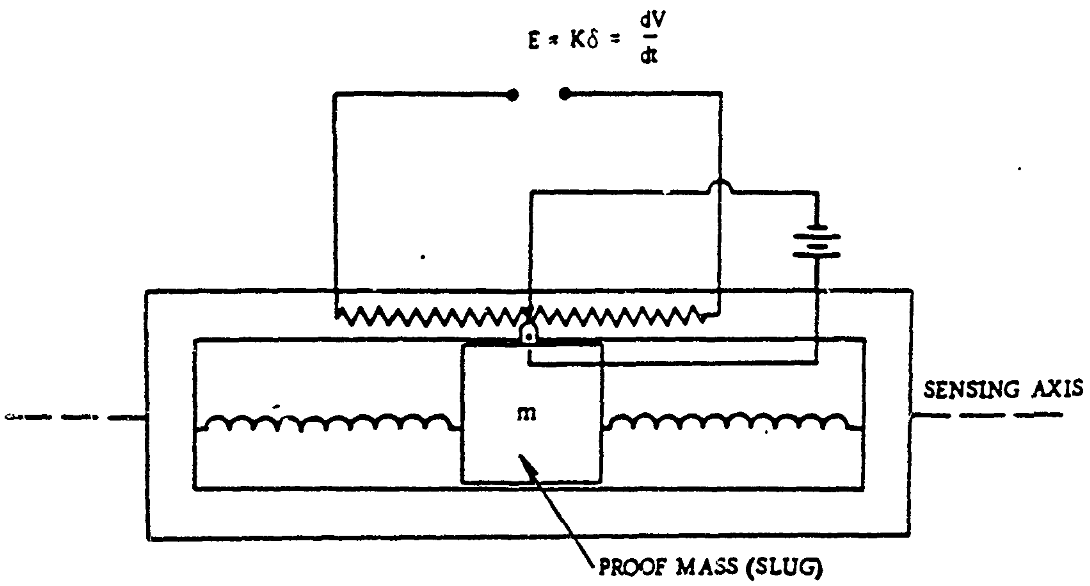
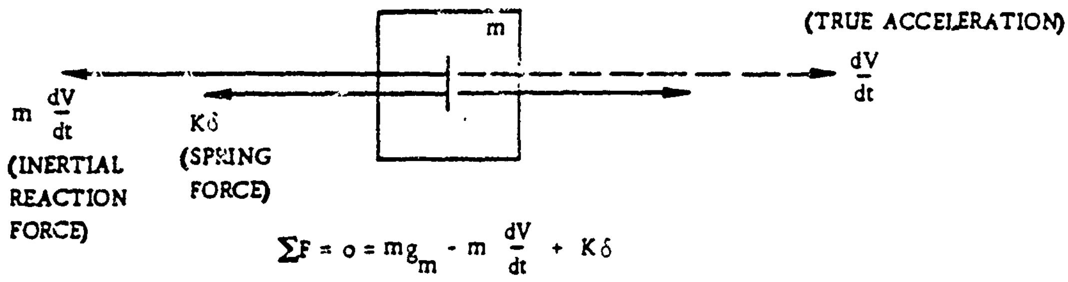


Figure 1. A stable platform using single-axis gyros.



(A) ACCELEROMETER (SCHEMATIC)



(B) SUMMATION OF FORCES

Figure 2. Concept of accelerometer behavior.

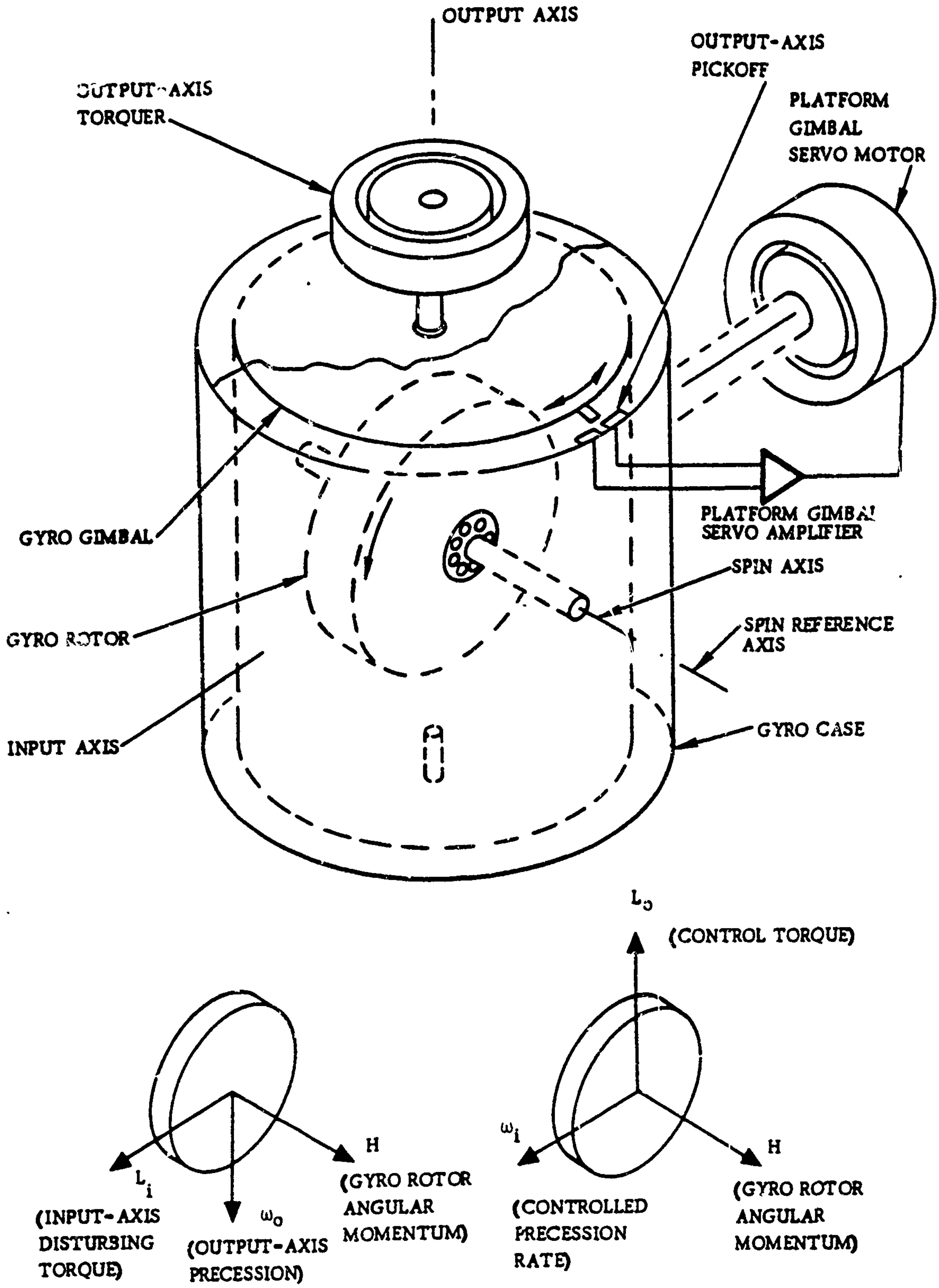


Figure 3. A single-axis gyro.

B. Gimbal Platforms Vs Strapdown Platforms

There are two widely used classes of inertial IMU's; gimballed and strapdown. In the gimballed class, the accelerometers are mounted on a stabilized platform which is maintained at the desired orientation by the attitude reference gyros and platform servo system. This gyro-stabilized IMU contains three accelerometers suspended by a gimbal assembly connecting the platform to the missile. The gimbal system provides angular isolation between the inertial instruments and the moving vehicle. The gyros sense any angular deviations of the platform from its desired orientation and provide signals to platform gimbal servos which restore the platform to the desired orientation. The accelerometers, physically mounted to the gyro platform, sense input accelerations along the gyro stabilized axes. The platform orientation and accelerometer orientations are continuously known, since the platform rotations are performed about an initially known platform orientation. Therefore, the accelerometer outputs can be integrated to determine horizontal and vertical velocity and position along these axes. Coordinate transformations can be used to determine velocity and positions in any other coordinate system.

Another class of platform is the "strapdown" platform. In a strapdown platform, the accelerometers are directly mounted or "strapped down" to the airframe and the components of acceleration at vehicle body coordinates are measured. The horizontal and vertical components of acceleration are then computed analytically using direction cosine matrices relating the body coordinates to the local level (North, East, Down) coordinates, or any other desired axis system.

C. Errors Within The System

The missile's inability to hit a target is caused by many factors, including inertial measurement errors, equation and computation errors, steering and cutoff errors, gravitational anomalies, reentry errors, and targeting errors.

Inertial measurement errors are those error sources which affect the inertial system's ability to accurately measure the velocity and position of the missile. These sources include the gyros, accelerometers, and the platform alignment errors.

Equation and computation errors may arise because of simplification of equations and numerical roundoff in an attempt to minimize computation time and storage requirements on the airborne computer.

Steering and cutoff errors are associated with the missile's ability to follow guidance commands and accurately cutoff engine thrust, if inertial type guidance is used.

Gravitational anomalies result from uncertainties in the knowledge of gravitational forces on the missile, and from errors in altitude.

Reentry errors arise from the abnormalities encountered when the missile reenters the atmosphere. These include nonstandard atmospheric densities and wind effects. Table 1 gives a summary of this type of error and shows relative magnitudes for the various error sources.

TABLE 1. Missile Re-Entry Errors

Errors	Magnitude Normalized	Comments
IMU	1	Cost and accuracy standard.
Steering and Cutoff	<.5	Dictates initial design.
Computation	.1	Sized proportional to IMU.
Gravitational	.1	Local anomalies greater than 60 arc-seconds.
Reentry	<.5	Large for short range PIA wind, target area atmosphere, small on guide-all-the-way.
Targeting	<.5	Can be minimized.

The majority of the error analysis in Table 1 deals with errors caused by the IMU. Errors are referred to as inertial platform constants and can be divided into the following three major groups:

1. Errors caused by imperfect accelerometer:

a. AKOX, Y, Z: Constant bias error, i.e., the accelerometer indicates some acceleration is being sensed for no acceleration input.

b. AKIX, Y, Z: Scale factor stability error caused by accelerometer along the input axis, e.g., a 1 g input acceleration causes the accelerometer to read 1.05 g's.

c. AKSX, Y, Z: Scale factor symmetry error caused by accleration along the input axis, i.e., a 1 g input in the positive direction causes a different accelerometer output magnitude.

d. AK2X, Y, Z: Second order nonlinearity error.

e. AK3X, Y, Z: Third order nonlinearity error.

f. AKPX, Y, Z: Bias sensitivity to cross-axis acceleration error caused by acceleration along the pendulous axis.

g. AKOPX, Y, Z: Bias sensitivity to cross-axis acceleration error caused by acceleration along output axis.

h. DXY, DXZ: Misalignment of x-accelerometer toward y, z.

i. DYX, DYZ: Misalignment of y-accelerometer toward x, z.

j. DZX, DZY: Misalignment of z-accelerometer toward x, y,

Figure 4 shows types of errors in the input-output characteristics of a real accelerometer.

2. Errors caused by imperfect gyroscopes:

a. DFX, Y, Z: Constant drift error.

b. DIX, Y, Z: Drift rate about X, Y, Z-axis caused by acceleration along input axis (e.g., mass unbalance along spin axis).

c. DSX, Y, Z: Drift rate about X, Y, Z-axis caused by acceleration along spin axis (e.g., mass unbalance along input axis).

d. DOPX, Y, Z: Drift rate about X, Y, Z-axis caused by acceleration along output axis (e.g., mass unbalance along spin axis).

e. ANSSX, Y, Z: X, Y, Z gyro error about X, Y, Z-axis caused by acceleration along spin axis due to nonlinear drift.

f. ANOOX, Y, Z: X, Y, Z gyro error about X, Y, Z-axis caused by acceleration along output axis due to nonlinear drift.

g. ANIIX, Y, Z: X, Y, Z gyro error about X, Y, Z-axis caused by acceleration along input axis due to nonlinear drift.

h. ANISX, Y, Z: Gyro error due to anisoelastic drift.

i. ANIOX, Y, Z: Gyro error due to anisoelastic drift.

j. ANSOX, Y, Z: Gyro error due to anisoelastic drift.

k. RANX, Y, Z: Gyro error due to random walk.

3. Errors caused by initial platform misalignment:

PHIX, Y, Z: Uncorrelated platform leveling error about X, Y, Z-axis.

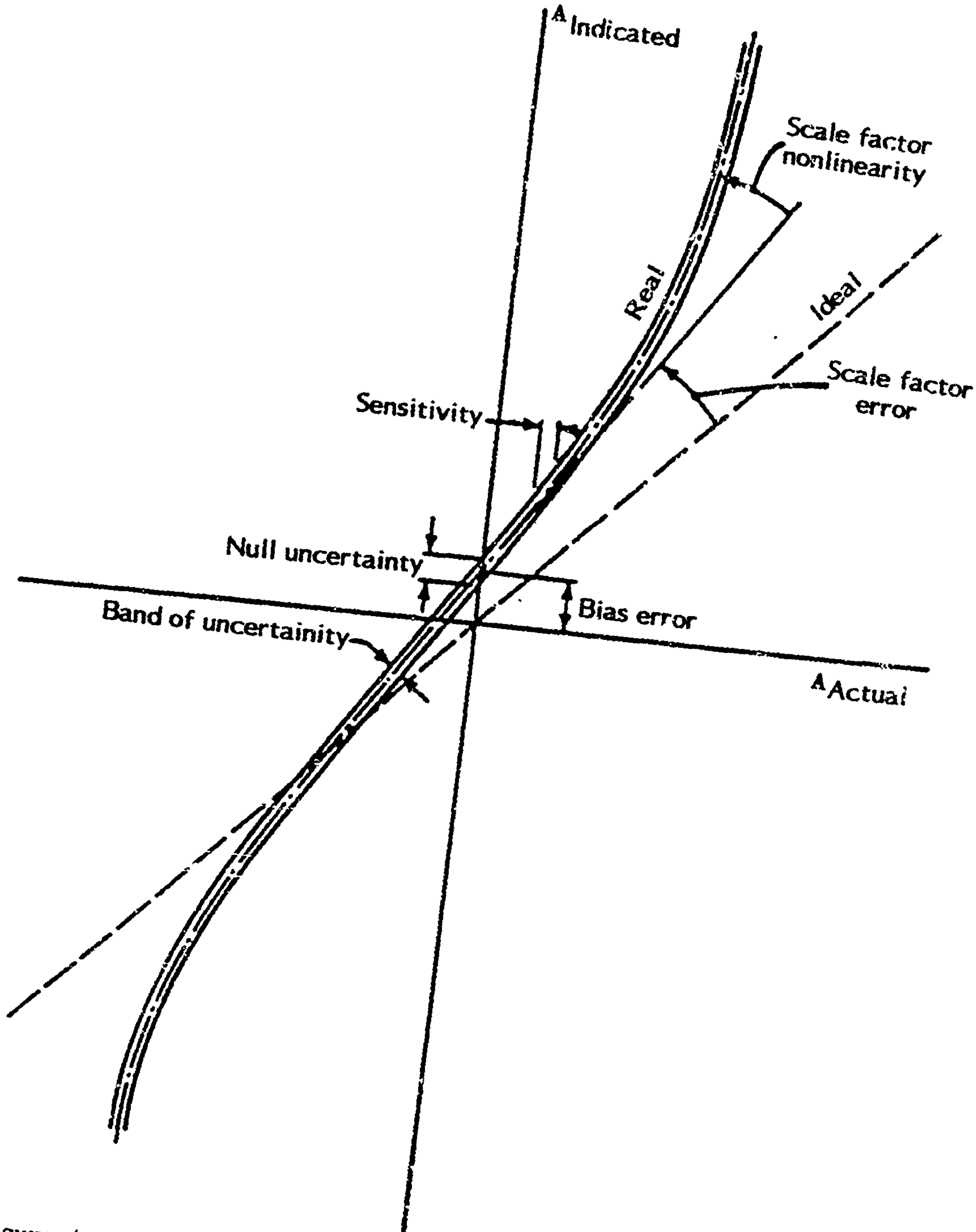


Figure 4. Types of errors in the input-output characteristics of a real accelerometer.

III. METHODOLOGY

A. Analysis Methods

1. Closed-Loop Error Analysis

Closed-loop error analysis methods for determining system errors caused by IMUs, motor performance, aero tolerances, computer quantization and initialization, wind, etc. for guided missile systems, require three distinct sets of trajectory data:

- Ideal (Reference)
- Actual (with error(s) included)
- Guidance (On-board)

The ideal vs the actual are compared to determine the performance error but the guidance data is required to close the guidance loop in the missile.

A missile flight simulation program (usually a six-degree-of-freedom computer program) is used to generate the three sets of trajectory data. The ideal is first generated based on no system errors and is identical to the actual for this condition.

When an error source is modelled into the flight-simulation program, it will cause the missile simulation to fly a perturbed trajectory; the actual and guidance trajectories may both be different from the ideal.

The actual and guidance trajectories may or may not be identical for a perturbed trajectory, depending on the error source causing the perturbation and on the type of missile guidance. The error sources may be placed into two categories.

a. Errors affecting navigation accuracy

- IMU
- Computer

b. Errors affecting vehicle performance

- Motor performance
- Aero tolerances
- Wind

For error sources in category a above, the actual and guidance trajectories will be different and for category b they will be the same. In either case, the system errors are determined as the differences between the ideal and actual trajectory data. Figure 5 shows this process.

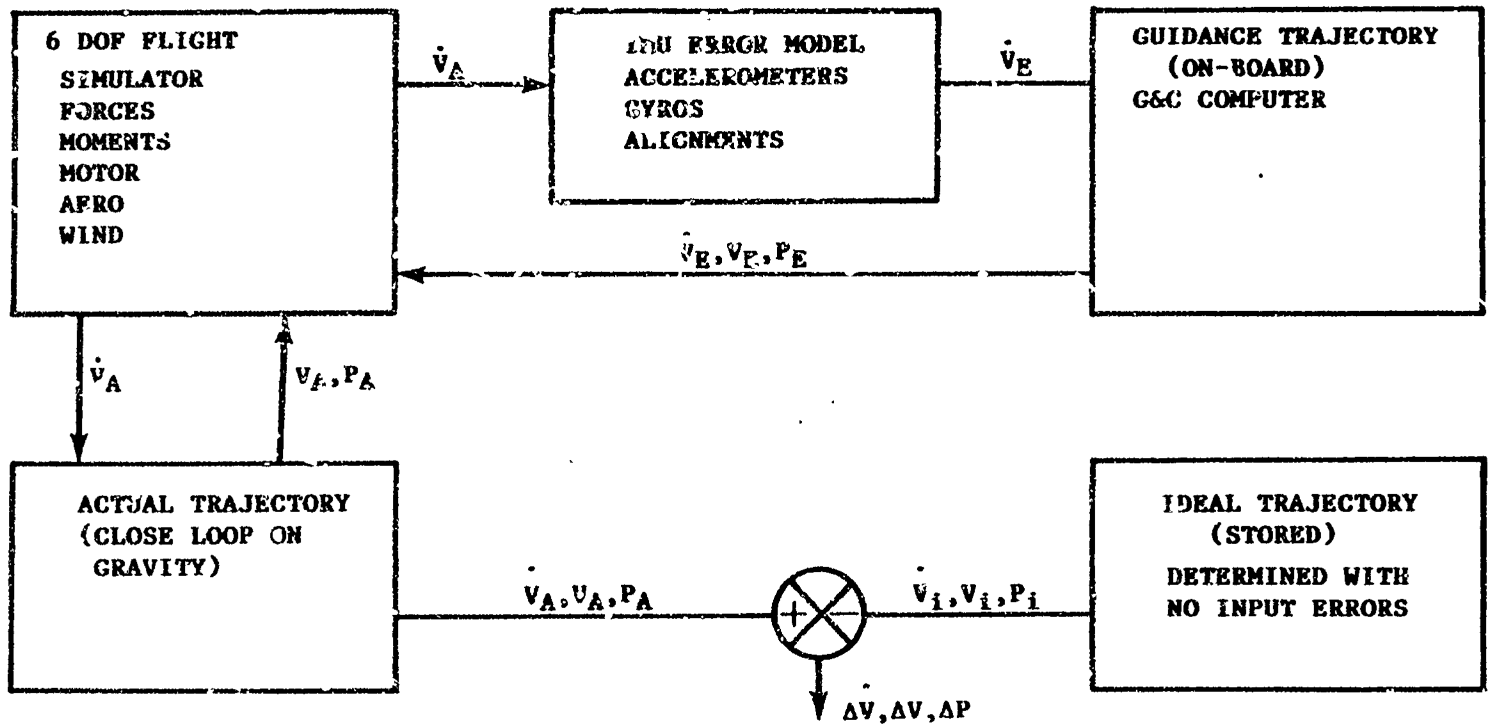


Figure 5. Closed-loop error analysis, block diagram.

2. Open-Loop Error Analysis

Error sources in category a above can be evaluated by open-loop error analysis methods, since it has been shown that for small error sources, the ideal and the guidance trajectories are nearly identical. The ideal trajectory can be regenerated by processing the ideal acceleration profile in a simplified computer program to generate ideal navigation data. This ideal acceleration profile data can be corrupted by IMU or computer errors and processed to generate a perturbed trajectory which can be compared to the ideal to determine IMU and computer errors (see Fig. 6). Error sources in category b above cannot be evaluated by this simplified method since they do not introduce on-board measurement errors.

B. Statistical Treatment of Errors

Generally, it is necessary to discuss guidance system accuracy on a probabilistic rather than an absolute basis. Prediction of absolute accuracy for a system depends on both the exact knowledge of each error coefficient and on knowledge of the error source behavior in the flight environment. Therefore, the system designer must rely on statistical methods to estimate system accuracy, since the factors which contribute to system error are usually imperfectly known. Some obvious effects which lead to uncertainties in the error sources include:

(1) Inability to measure the value of error coefficients to the required precision in the presence of noise.

(2) Changes in error coefficients from the time of measurement until launch, and sometimes during flight.

(3) Changes in environmental conditions such as temperature and pressure.

(4) Vibrational effects during flight.

To begin the statistical evaluation of guidance system accuracy, one must first determine the statistical parameters which describe each of the error sources. Typically, the average values and the standard deviations or root-mean-square values of the individual errors are pertinent statistical parameters. To determine the statistical parameters, tests may be made on a suitable population of components and the anticipated effects of environmental changes such as temperature and vibration on each error coefficient may be calculated.

To determine system error on a probabilistic basis through combination of individual error coefficients, the statistical parameters must first be reduced to a common base. The base may be the velocity and position errors at thrust termination or the velocity and position errors at target impact. By application of the central limit theorem, the final errors resulting from each individual error source can be combined to give the expected system error. The central limit theorem states that the sum of a large number of variables is approximately normally or Gaussian distributed regardless of the

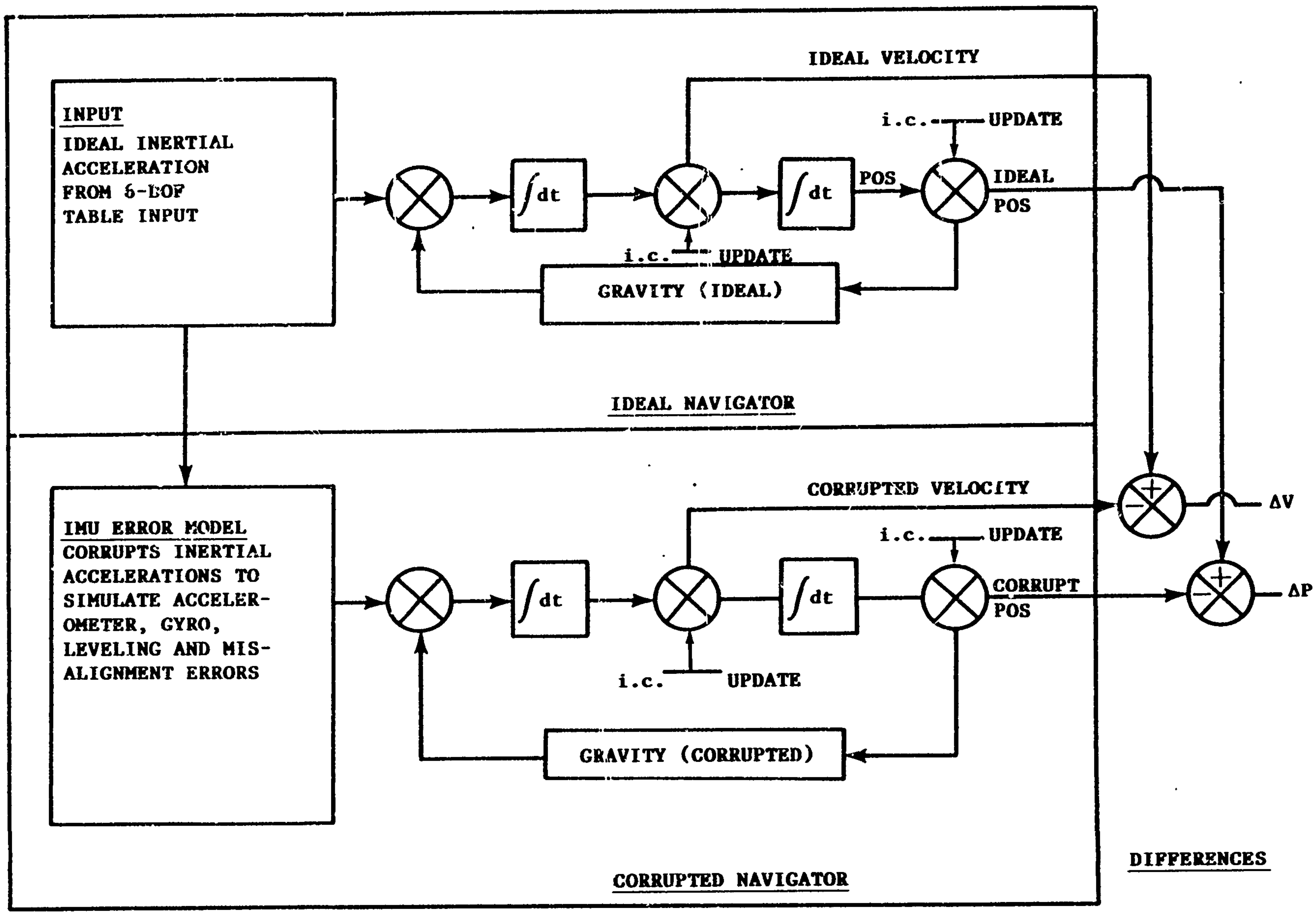


Figure 6. Open-loop error analysis, block diagram.

particular distributions of the individual components of the sum. The standard deviation of the sum, sigma, is the RSS value of the standard deviations of the individual components.

Two basic methods of evaluating position and velocity errors are used in the IMU error analysis program. These are the Monte Carlo method and the Root-Sum-Square (RSS) method. If the individual IMU error sources are not correlated, the standard deviations generated by the Monte Carlo techniques should converge the RSS standard deviations, and the mean values generated by the Monte Carlo technique should be zero. This was shown empirically in the validation of the program and is graphically illustrated in Figures 7 and 8, respectively.

The Monte Carlo method requires a large number of trajectory runs, each corrupted by a "typical," randomly configured platform. Each run is unique in that each of the N error sources is a randomly selected function of the 1 sigma manufacturing tolerance. Thus, each trajectory run evaluates a "typical" IMU with all the associated error sources. The statistical data (standard deviations and means) are then generated for a large sampling of runs (100-200).

The RSS method requires a specific trajectory run for each of the "N" IMU error sources. The results of the N trajectory runs are then root-sum-squared to give the standard deviations. Figure 7 illustrates data processing for the Monte Carlo and for the RSS methods.

1. Monte Carlo Method

The Monte Carlo method is the first method of error analysis used. All Monte Carlo computer calculations are based on the generation and use of a set of random numbers, from a subroutine random number generator. The subroutine generates numbers having a bell-shaped Gaussian ("normal") distribution. Statistically, the total area under a Gaussian curve is one square unit. Therefore, the area between any two points is the proportion of cases which lie between the two points. Choosing a value of one standard deviation on either side of the mean, the probability of finding the function within this area is 68 percent. When all the N inertial platform coefficients are input, by a Gaussian random number generator for each run, a typical inertial platform which falls within a family described by the manufacturing tolerances is represented. With this method, each run has an ideal trajectory which is compared to a trajectory corrupted with all N inertial platform errors that were input. These input errors are generated by a random number generator program obtained from the Oak Ridge Laboratory. Several Monte Carlo runs are made in succession. The confidence level of the results obtained increases as the number of Monte Carlo runs made increases. The confidence level is proportional to the inverse of the square root of the number of runs. Using the open-loop error analysis, 100 runs gives an estimate of standard deviation to within 10 percent. At the end of all 100 of the Monte Carlo runs, the means and the standard deviations of the position errors are calculated. The circular error probability (CEP) is calculated. The CEP is a measure of the guidance system accuracy, is defined as the radius of a circle around the target, and has a 50 percent probability that the guidance system will guide the vehicle into the circle.

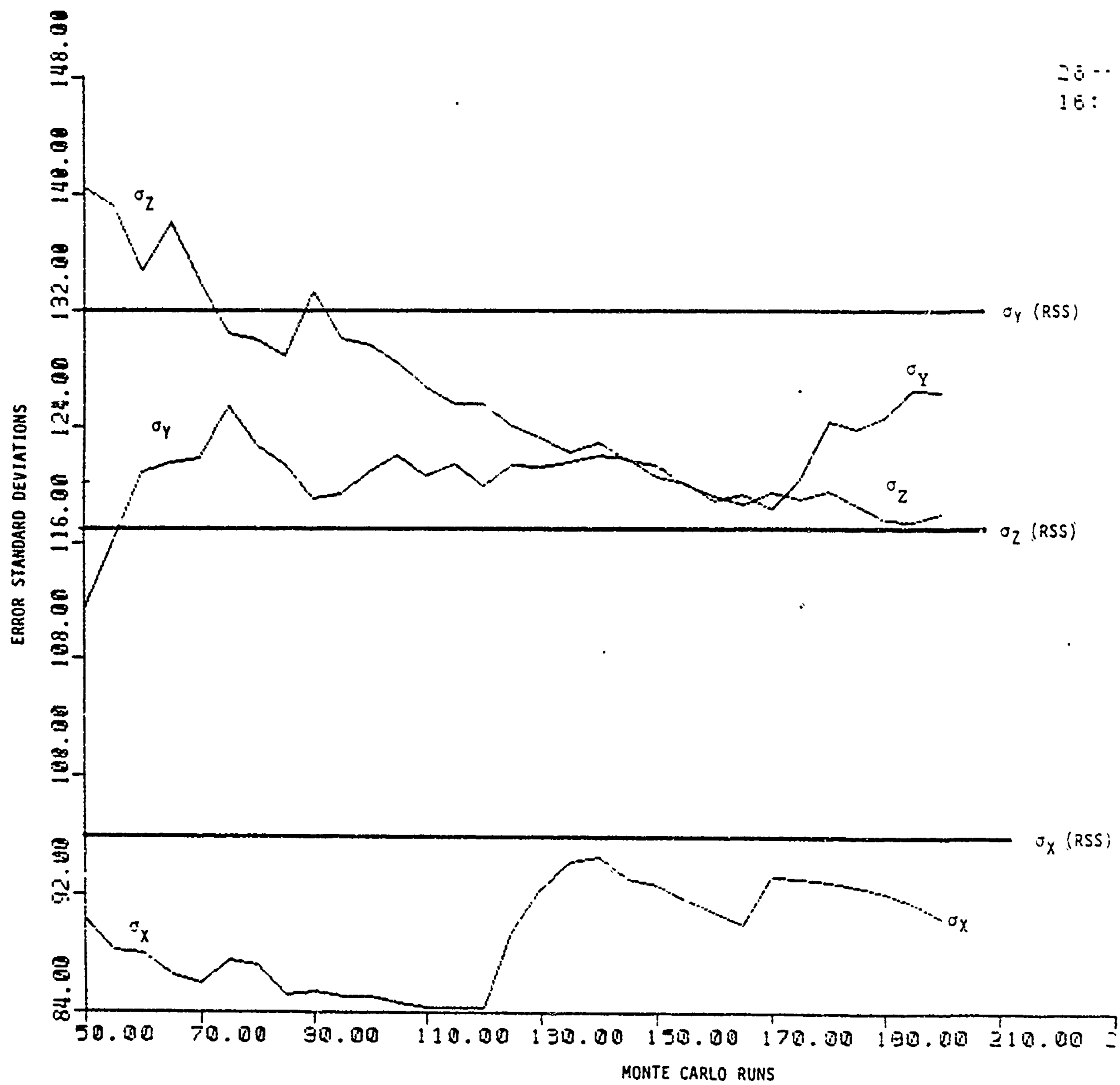


Figure 7. Cumulative standard deviations as function of number of Monte Carlo runs.

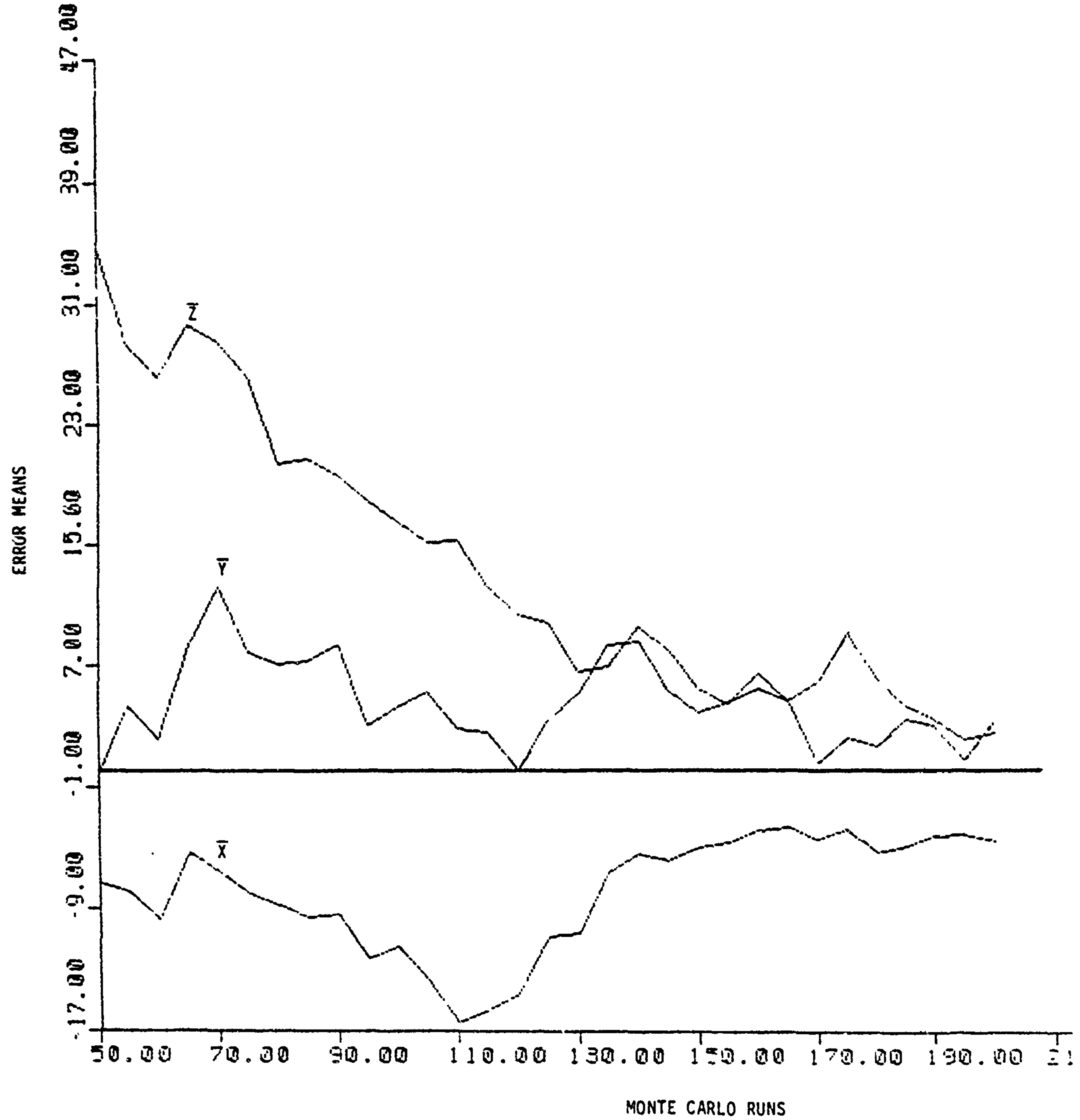


Figure 8. Cumulative means as function of number of Monte Carlo runs.

2. Root-Sum-Square (RSS) Method

The RSS method is the second method used in the error analysis program. This procedure is as follows: First, the program is run with an ideal trajectory in which the inertial platform coefficients have zero error values. This nominal trajectory will then be used as a reference by which the corrupted trajectory is compared. Corrupted trajectories are then run to compare their results with the results obtained from the ideal trajectory. The first corrupted trajectory will have had one inertial platform coefficient error input. All other inertial platform coefficients will have input values of zero. A unique set of x, y, z position errors is produced from the corrupted trajectory. The error values will have a standard deviation. Now, a comparison is made between the results obtained from the ideal trajectory and the results obtained from the corrupted trajectory. This enables calculation of how much each inertial platform constant contributes to the deviation from the corrupted trajectory to the ideal trajectory. The most pertinent parameters used in this comparison are those of position, since a guidance system's accuracy is evaluated in terms of the missile's ability to intercept the target position. Since there are N inertial platform constants, the trajectory is run N different times, each with an input of one inertial platform error and then each is compared to the ideal trajectory. As many as 70 different error sources have been input. The effect each error has on the trajectory can be observed individually or can be observed as an overall effect by taking the RSS of the position errors. The RSS of the errors is representative of the system. The RSS is found by taking the square root of the sum of the errors squared, as shown in the following equation:

$$X = \sqrt{\Delta X1^2 + \Delta X2^2 + \ldots \Delta XN^2}$$

IV. VALIDATION RESULTS

A. Normalized Integrals

One general error analysis procedure commonly used for relating measurement error sources to velocity and position errors involves using normalized integrals of acceleration. Examination of these equations reveals that most integrals contain one acceleration component or a cross product of two components. Only a few normalized integrals need to be evaluated for analysis of a three-accelerometer system since there is a repetitious pattern. A complete set of normalized integrals is quantitatively evaluated over a desired standard trajectory. Then each error coefficient is multiplied times the appropriate integral value to obtain the effective velocity error. Position errors can be evaluated by another integration. These results compare to the open-loop analysis described below. The closed-gravitational loop is not taken intc account with the normalized integral method. Therefore, this method becomes less accurate with longer flights. This analytical method is quick, but less accurate than the flight simulation method of analysis. The following is the procedure for use of normalized integrals:

Using the accelerometer bias error, write the equation

$$\ddot{x} = \ddot{x}_i + B_x \tag{1}$$

where $\ddot{x}_i$ = ideal x acceleration and B_x = bias error.

Integrating

$$\int \ddot{x}dt = \int (\ddot{x}_i + B_x)dt \tag{2}$$

$$\dot{x} = \dot{x}_i + B_x t$$

$$\dot{x} - \dot{x}_i = \dot{x}_E = B_x t$$

where $\dot{x}_E$ = x velocity error.

Integrating

$$\int \dot{x}dt = \int (\dot{x}_i + B_x t)dt \tag{3}$$

$$x = x_i + B_x\frac{t^2}{2}$$

$$x - x_i = x_E = B_x\frac{t^2}{2}$$

where x_E = x position error.

Verify these equations with the values given on page 307 of Reference 2.

$$\dot{x}_E = (2 \times 10^{-5} g)(32.2 \text{ ft/s}^2)(300 \text{ s}) = .19 \text{ ft/s} \tag{4}$$

$\dot{x}_E$ = .19 ft/s book value.

After the equations are verified, use a normalized integral on the 100 km trajectory at time of cut off (80 s)

$$\dot{x}_E = B_x t \tag{5}$$

$$\dot{x}_E = (111.8\ \mu g) \left(1 \times 10^{-6} \frac{g}{\mu g}\right) \left(\frac{9.8 \text{ m/s}^2}{1 \text{ g}}\right) \left(80 \text{ s}\right)$$

$$\dot{x}_E = .087 \text{ m/s}$$

$$X_E = \frac{B_x t^2}{2} = (111.8\ \mu g) \left(1 \times 10^{-6} \frac{g}{\mu g}\right) \left(\frac{9.8 \text{ m/s}^2}{1 \text{ g}}\right) \left(\frac{80 \text{ s}}{2}\right)^2$$

$$X_E = 3.5 \text{ m}$$

Compare the values obtained by normalized integrals above to the values obtained in the open-loop error analysis simulation at 80 s $\dot{x}_E$ = .08 m/s

$$\left(\frac{.087 - .086}{.087}\right) (100) = 1\% \text{ difference,} \tag{6}$$

simulation at 80 s X_E = 3.47 m

$$\left(\frac{3.5 - 3.47}{3.5}\right) (100) = .86\% \text{ difference.} \tag{7}$$

B. Open-Loop Error Analysis Program

The first step in validation of the open-loop error analysis program is to generate an acceleration profile whose trajectory will closely match the trajectory found in the Singer Kearfott GEAP document (Generalized Error Analysis Program) (see Reference 5). Comparison runs are made between the open-loop error analysis program and the reference document. Because the trajectories are closely matched, the position errors and CEP between the reference trajectory and the open-loop error analysis program trajectory should closely match, given the same initial conditions (i.e., give the same input of platform error coefficients).

In order to closely match the trajectories, an acceleration profile is obtained from the GEAP document and incorporated into the error analysis program. The acceleration profile gives correlated values of time, x acceleration, y acceleration, and z acceleration. The value of missile velocity is obtained by integration of acceleration, and the value of missile position is obtained by subsequent integration of this velocity. Some refinements must be made on the acceleration profile incorporated into the openloop program for its trajectory to be comparable to the Singer Kearfott program. Modifications to the acceleration profile are made at burn-out, second stage ignition, and re-entry, for the error analysis trajectory to follow the same trajectory as the Singer Kearfott program.

The first acceleration profile found in the GEAP Document is a normal range trajectory which travels 742 km in 482 s. The open-loop error analysis acceleration profile is generated to closely match this trajectory. The vehicle travels 760 km in 481 s. Position errors are calculated for each individual error coefficient input and then the RSS of all the position errors is calculated. Because the trajectories are closely matched, comparable results are obtained between the open-loop error analysis results and the results found in the GEAP document. These results are tabulated in Table 2.

The second acceleration profile found in the GEAP document is a long range trajectory. Comparable results between the two error analysis programs are tabulated in Table 3 and indicate that the open-loop error analysis program is running properly.

The results from this program compared favorably with the results from those discussed in Reference 5.

C. Comparison Runs Between the Monte Carlo Method and the Root-Sum-Square Method

The next step in validation of the open-loop error analysis program is to favorably compare the results obtained from the Monte Carlo Method and the results obtained from the RSS Method, as theory indicates. These results are tabulated in Table 4 for generic trajectories. The graph in Figure 9 shows that over an increasing number of Monte Carlo runs, the position errors obtained by the Monte Carlo runs approach the position errors obtained by the RSS Method. Figure 10 shows that the position means approach zero over an increasing number of runs.

D. Comparison of the Monte Carlo Method and the RSS Method with the Error-Covariance Method

Another method of validating the open-loop-error analysis program is to compare the Monte Carlo and the RSS results with the results obtained in an error covariance method. The error covariance method involves a study of the dynamics of the error itself and can be seen in Reference 4. As shown in the paper in Reference 4, the results of the open-loop error analysis program and the error covariance program are in close agreement.

TABLE 2. Comparison Results of the Normal Range Trajectory

ERROR	GEAP REFERENCE POSITION ERRORS (meters) DX	DY	DZ	OPEN-LOOP ERROR ANALYSIS PROGRAM POSITION ERRORS (meters) DX	DY	DZ
AKOX = 112 μg	59	0	.3	53	0	.4
AKOY = 112 μg	0	60	0	0	52	0
AKOZ = 112 μg	.5	0	59	.4	0	58
AKIX = 97 ppm	73	0	.7	74	0	.6
AKIY = 97 ppm	0	0	0	0	0	0
AKIZ = 48 ppm	.5	0	52	.4	0	55
AKSX = 70 ppm	54	0	3	53	0	.4
AKSY = 70 ppm	0	0	0	0	0	0
AKSZ = 70 ppm	.7	0	30	.6	0	30
AK2X = 20 $\mu g/g^2$	56	0	.5	56	0	.5
AK2Y = 20 $\mu g/g^2$	0	0	0	0	0	0
AK2Z = 15 $\mu g/g^2$	.5	0	59	.4	0	59
AK3X = 1 $\mu g/g^3$	11	0	.1	12	0	.1
AK3Y = 1 $\mu g/g^3$	0	0	0	0	0	0
AK3Z = 1 $\mu g/g^3$	.1	0	15	.10	0	15
DZY = 41 arc-sec	0	.06	0	0	0	0
DZX = 31 arc-sec	1	0	128	1	0	130
DYX = 17 arc-sec	0	68	0	0	63	0
DFX = .02 °/hr	0	3	0	0	4	0
DFY = .02 °/hr	0	0	3	4	0	3
DFZ = .08 °/hr	3	13	0	0	14	0
DIX = .09 °/hr/g	0	30	0	0	30	0
DIY = .09 °/hr/g	0	30	0	0	0	0
DIZ = .09 °/hr/g	0	48	0	0	49	0
DSX = .04 °/hr/g	23	23	0	0	23	0
DSY = .04 °/hr/g	0	0	24	23	0	25
DSZ = .04 °/hr/g	25	13	0	0	14	0
DOPX = .08 °/hr/g	25	0	28	0	0	0
DOPY = .08 °/hr/g	0	0	28	26	0	29
DOPZ = .08 °/hr/g	0	0	0	0	0	0
ANSSX = .0 $°/hr/g^2$	5	5	0	0	5	0
ANSSY = .0 $°/hr/g^2$	0	0	5	5	0	6
ANSSZ = .0 $°/hr/g^2$	0	3	0	0	3	0
ANOOX = .0 $°/hr/g^2$	3	0	0	0	0	0
ANOOY = .0 $°/hr/g^2$	0	35	3	3	0	3
ANOOZ = .0 $°/hr/g^2$	0	0	0	0	0	0
ANISX = .0 $°/hr/g^2$	0	36	0	0	37	0
ANISY = .0 $°/hr/g^2$	0	36	0	0	0	0
ANISZ = .0 $°/hr/g^2$	0	3	0	0	37	0
ANIIX = .0 $°/hr/g^2$	0	5	0	0	3	0
ANIIY = .0 $°/hr/g^2$	0	5	0	0	0	0
ANIIZ = .0 $°/hr/g^2$	0	0	0	0	5	0
ANIOX = .0 $°/hr/g^2$	0	0	0	0	0	0
ANIOY = .0 $°/hr/g^2$	0	0	0	0	0	0
ANIOZ = .0 $°/hr/g^2$	0	0	0	0	0	0
ANSOX = .09 $°/hr/g^2$	9	0	10	0	0	0
ANSOY = .09 $°/hr/g^2$	9	0	10	9	0	10
ANSOZ = .09 $°/hr/g^2$	0	0	0	0	0	10
PHIX = 3 arc-sec	0	16	0	0	17	0
PHIY = 3 arc-sec	16	0	14	16	0	14
PHIZ = 57 arc-sec	0	207	0	0	210	0
ROOT-SUM SQUARE	118	259	184	126	241	188
CEP = 218				CEP = 213		

TABLE 3. Comparison Results of the Maximum Range Trajectory

ERROR	GEAP REFERENCE POSITION ERRORS (meters) DX	DY	DZ	OPEN-LOOP ERROR ANALYSIS PROGRAM POSITION ERRORS (meters) DX	DY	DZ
AKOX = 112 μg	167	2	262	81*	.2	4
AKOY = 112 μg	3	177	66	.2	30	1
AKOZ = 112 μg	2	1	23	5	1	113
AKIX = 97 ppm	170	.4	10	174	4	9
AKIY = 97 ppm	105	3	.6	.1	44	.6
AKIZ = 48 ppm	7	2	153	6	2	2
AKSX = 70 ppm	126	.3	7	126	.3	7
AKSY = 70 ppm	.08	31	.4	0	31	.4
AKSZ = 70 ppm	10	2	225	9	2	2
AK2X = 20 g/g^2	182	.4	10	189	.4	10
AK2Y = 20 μg/g^2	.03	11	.1	0	12	.2
AK2Z = 15 μg/g^2	10	2	222	9	2	228
AK3X = 1 μg/g^3	58	.1	3	63	.2	3
AK3Y = 1 μg/g^3	0	.9	.01	0	1	0
AK3Z = 1 μg/g^3	4	.8	92	3	.1	94
DZY = 41 arc-sec	6	2	122	5	1	124
DZX = 31 arc-sec	17	4	364	16	4	372
DYX = 17 arc-sec	0	143	2	.4	147	2
DFX = .02 °/hr	.1	12	3	.2	13	3
DFY = .02 °/hr	12	.1	11	12	.1	12
DFZ = .09 °/hr	11	42	.1	11	44	.0
DIX = .09 °/hr/g	27	108	.5	2	113	38
DIY = .09 °/hr/g	27	108	.5	27	.3	35
DIZ = .10 °/hr/g	40	156	.3	41	163	.1
DSX = .04 °/hr/g	.9	79	23	1	81	26
DSY = .04 °/hr/g	75	.8	91	78	.9	96
DSZ = .04 °/hr/g	12	43	.1	12	48	0
DOPX = .08 °/hr/g	88	22	120	.4	24	8
DOPY = .08 °/hr/g	88	22	120	91	1	118
DOPZ = .08 °/hr/g	5	21	0	6	23	0
ANSSX = .0 °/hr/g^2	.2	20	6	.3	21	7
ANSSY = .0 °/hr/g^2	17	.2	23	20	.2	25
ANSSZ = .0 °/hr/g^2	3	11	0	3	12	0
ANOOX = .0 °/hr/g^2	.01	.8	.2	0	.8	.3
ANOOY = .0 °/hr/g^2	12	.1	15	13	.2	16
ANOOZ = .0 °/hr/g^2	.2	.7	0	.2	.8	0
ANISX = .03 °/hr/g^2	38	151	.6	3	161	57
ANISY = .03 °/hr/g^2	38	151	.6	39	.4	52
ANISZ = .03 °/hr/g^2	38	148	.2	40	157	.1
ANIIX = .0 °/hr/g^2	.9	12	3	.2	13	4
ANIIY = .0 °/hr/g^2	.9	12	3	.8	.1	1
ANIIZ = .0 °/hr/g^2	4	18	0	5	2	0
ANIOX = .0 °/hr/g^2	3	3	3	0	3	1
ANIOY = .0 °/hr/g^2	3	3	3	3	.0	4
ANIOZ = .0 °/hr/g^2	.9	3	0	.9	4	0
ANSOX = .09 °/hr/g^2	36	9	52	.2	10	4
ANSOY = .09 °/hr/g^2	36	9	52	38	.5	52
ANSOZ = .09 °/hr/g^2	2	8	0	2	8	0
PHIX = 3 arc-sec	.4	38	10	.5	39	11
PHIY = 3 arc-sec	36	.3	38	37	.3	39
PHIZ = 57 arc-sec	122	482	.8	124	492	.1
ROOT-SUM SQUARE	357	667	556	363	614	584
CEP = 594				CEP = 568		

*AKOX,Y,Z Not calculated during mid-course.

TABLE 4. IMU Error Analysis Results*

Error Analysis	σx (m)	σy (m)	$\sigma 2$ (m)	$\overline{B}x$ (m)	$\overline{B}y$ (m)	$\overline{B}z$ (m)	CEP (m)
<u>Medium Range</u>							
Open-Loop,							
RSS Method	126	241	188	--	--	--	216
Monte Carlo							
25 runs	134	253	175	-25	-30	2	227
50 runs	124	232	163	-21	-15	-5	208
100 runs	133	230	176	-10	-34	-2	213
Reference (SKD)	118	259	184	--	--	--	220
<u>Long Range</u>							
Open-Loop,							
RSS Method	361	613	577	--	--	--	574
Monte Carlo							
25 runs	397	676	515	-46	-92	-5	629
50 runs	373	604	499	-28	-66	-17	565
100 runs	385	589	533	1	-108	15	573
Reference (SKD)	357	667	556	--	--	--	603

*MICOM OLEA PROGRAM

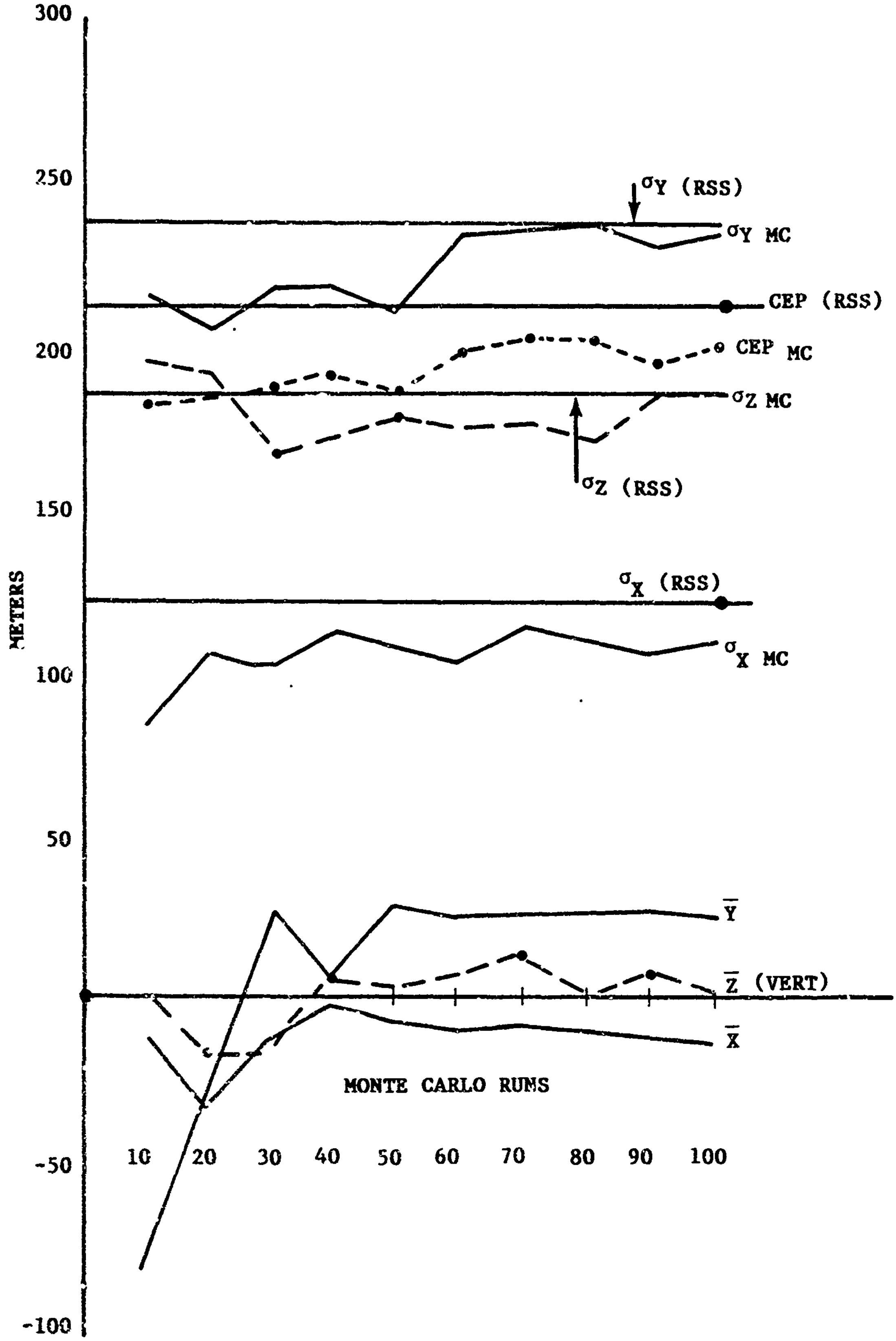


Figure 9. Comparison of Monte Carlo vs RSS medium range PII IMU errors.

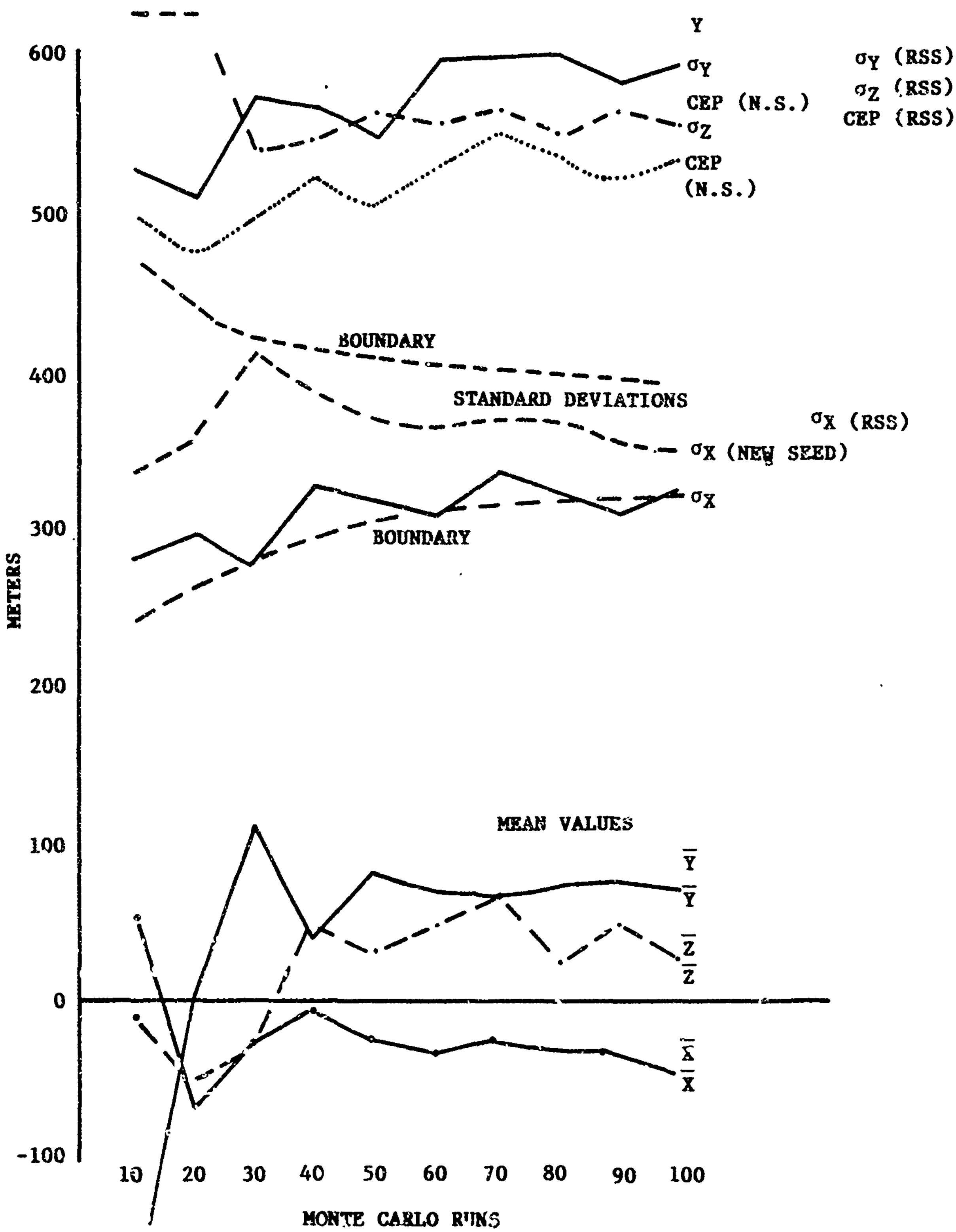


Figure 10. Comparison of Monte Carlo vs RSS of long range PII IPA errors.

V. SUMMARY

In summary, this report covers two basic open-loop methods of analyzing errors within an inertial platform. These two methods are Monte Carlo and the RSS method. The results from both of these methods were compared with the error-covariance method of inertial platform analysis. All three methods give comparable results. Additionally, all three methods have advantages and disadvantages. The "error-covariance" method is the most economical, requiring only one computer trajectory run, but provides the least insight into the physics of the problem. The "Root-Sum-Square" method requires one computer trajectory run for each individual IMU error source and provides deterministic effects for each error source which can be reconsolidated easily. It does not, however, provide the physical insight into the problem that the "Monte Carlo" method does. The "Monte Carlo" is the least economical of the 3 methods, typically requiring 100 or more simulated trajectories for an IMU with 20-30 independent error sources. It provides great insight into the process, since the assumed IMU in each of the 100+ trajectories has a randomly selected set of errors which fit in the family of IMU's as defined by the manufacturing specifications.

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